CALCULATION OF THE BLAST EFFECT IN BRITTLE ROCK (FRACTURING BY CRUSHING, FORMATION OF CLEAVAGE CRACKS, AND SEPARATION)

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An algorithm is presented for calculating the dynamics of the development of a blast in solid, brittle fracturing rocks. The stages of the phenomenon arising and alternating in various sequences depending on the mechanical properties of the rock and power of the blast are examined in detail.

1. Let there be a spherical cavity of radius r_0 in a space filled with isotropic brittle rock. The medium is at rest and compressed by hydrostatic pressure P_h. The cavity is filled with an explosive charge which, after detonation, is converted to gas with initial pressure P_0. It is required to determine the character of fracture, volume of fractured rock, parameters of the waves radiated by the seat of the blast, etc. as a function of the properties of the rock, explosive, and initial hydrostatic pressure.

The general approach to the solution of such problems is given in [1]. One of the properties of nonporous brittle solid rocks is that on reaching the strength condition fracture can be of two types: fracture with the formation of numerous separation cracks oriented normal to the fracture front and fracture with the formation of numerous cleavage cracks dividing the rocks into small blocks. In porous rocks fracture can occur with all-around compression owing to fracture of the brittle porous skeleton.

We will consider regions of rock involved in movement; we will assume at first that the material is unfractured and then is in a fractured state after the effect of various fracturing mechanisms.

Unfractured Region. It is considered that the unfractured material is described by a linear elastic model. The solution of the fundamental equations for this region in the case of central symmetry is given by the equations [1]

$$\begin{gathered}
\sigma_r = -\rho c_0^2\left\{\frac{f''(\zeta)}{x} + \frac{2(1-2\sigma)}{1-\sigma}\left[\frac{f'(\zeta)}{x^2} + \frac{f(\zeta)}{x^3}\right]\right\} - P_h \\
\sigma_\varphi = \sigma_\theta = -\rho c_0^2\left\{\frac{\sigma}{1-\sigma}\,\frac{f''(\zeta)}{x} - \frac{1-2\sigma}{1-\sigma}\left[\frac{f'(\zeta)}{x^2} + \frac{f(\zeta)}{x^3}\right]\right\} - P_h \\
V = c_0\left[\frac{f''(\zeta)}{x} + \frac{f'(\zeta)}{x^2}\right], \quad u = r_0\left[\frac{f'(\zeta)}{x} + \frac{f(\zeta)}{x^2}\right] \\
\zeta = \tau - x, \quad x = r/r_0, \quad \tau = c_0 t/r_0
\end{gathered} \tag{1.1}$$

Here x, τ are dimensionless coordinates, c_0 is the velocity of the longitudinal elastic waves in the unfractured region, t is time, r is a Lagrangian coordinate, ρ is the initial density of the medium, σ_r, σ_θ, σ_φ are stresses on the coordinate areas, V is the velocity of particles in a radial direction, u is radial displacement, σ is Poisson's ratio.

Region of Fracture by Separation. It is considered that the material is divided into elastic conical bars which withstand only radial stress, and the hoop stress is equal to zero in the entire region [1]. In this case

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$$\sigma_r = -\rho c_0^2 \lambda^2 \left[\frac{f_1^{\cdot}(\xi) + f_2^{\cdot}(\eta)}{x} + \frac{f_1(\xi) + f_2(\eta)}{x^2}\right],$$
$$\lambda = \frac{c_1}{c_0} = \left(\frac{(1-2\sigma)(1+\sigma)}{1-\sigma}\right)^{1/2}$$
$$\sigma_\theta = \sigma_\varphi = 0, \quad V = \frac{c_0\lambda}{x}\,[f_0^{\cdot}(\xi) + f_2^{\cdot}(\eta)], \quad c_1 = \left(\frac{E}{\rho}\right)^{1/2} \tag{1.2}$$
$$u = \frac{r_0}{x}\left[f_1(\xi) + f_2(\eta) + \frac{1-\sigma}{1+\sigma}\, p_h x^2\right], \quad p_h \equiv \frac{P_h}{\rho c_0^2} \begin{pmatrix} \xi = \lambda\tau - x \\ \eta = \lambda\tau + x \end{pmatrix}$$

Here c_1 is the velocity of elastic waves in material fractured by radial cracks, E is Young's modulus.

Region of Fracture by Cleavage. It is considered that the material is divided by cleavage cracks and described by Hooke's law for dilitational strain

$$\frac{1}{3}(\sigma_r + 2\sigma_\theta) = \rho c_0^2 q^2 \left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) - P_h$$
$$q = \frac{c_2}{c_0} = \left(\frac{1+\sigma}{3(1-\sigma)}\right)^{1/2}, \quad c_2 = \left(\frac{E}{3\rho(1-2\sigma)}\right)^{1/2} \tag{1.3}$$

and by the condition of plasticity

$$\sigma_r - \sigma_\theta = -2\tau_{*1} \tag{1.4}$$

Here c_2 is the velocity of sound in material fractured by cleavage cracks; τ_{*1} characterizes friction on the surface of the cleavage cracks. The solution is given by the formulas

$$\sigma_r = -\rho c_0^2 \frac{q^2}{x}\,[F_1^{\cdot\cdot}(\xi_1) + F_2^{\cdot\cdot}(\eta_1)] - 4\tau_{*1}\ln x + P_h, \quad \sigma_\theta = \sigma_r + 2\tau_{*1}$$
$$V = c_0 q\left[\frac{F_1^{\cdot\cdot}(\xi_1) - F_2^{\cdot\cdot}(\eta_1)}{x} + \frac{F_1^{\cdot}(\xi_1) + F_2^{\cdot}(\eta_1)}{x}\right]$$
$$u = r_0\left[\frac{F_1^{\cdot}(\xi_1) - F_2^{\cdot}(\eta_1)}{x} + \frac{F_1(\xi_1) + F_2(\eta_1)}{x^2} + \frac{4(1-\sigma)}{1+\sigma}\, T_{*1} x \ln x\right] \tag{1.5}$$
$$T_{*1} \equiv \frac{\tau_{*1}}{\rho c_0^2}, \quad \xi_1 = q\tau - x, \quad \eta_1 = q\tau + x$$

Region of Fracture by Crushing. It is considered that the crushed material in plastic flow is described by equations of a continuous meaium, by the equation of motion

$$\rho_* \frac{dV}{dt} = \frac{\partial \sigma_r}{\partial r} + \frac{2}{r}(\sigma_r - \sigma_\theta) \tag{1.6}$$

by the equation of continuity, which with consideration of the condition of incompressibility of the medium adopted here allows a general solution of the form

$$V = c(t) / r^2 \tag{1.7}$$

where c(t) is an arbitrary function, and by the condition of plasticity which is adopted by analogy with the case of soft ground [2] in the form

$$\sigma_\theta = a\sigma_r + b \tag{1.8}$$

Here ρ_* is the density of the crushed material. Henceforth we will assume for simplification of the mathematical procedure* that upon crushing the material acquires maximum density, determined by the formula

$$\rho_* = k\rho, \quad k = \text{const} \tag{1.9}$$

In Eqs. (1.8) and (1.9) a and b are parameters dependent on the medium and on the hydrostatic pressure (P_h), and k is the coefficient of maximum compaction [3].

*We note that consideration of the variability of compaction of the material at the front of the shock wave does not cause fundamental difficulties, but the equations and formulas of the final mathematical problem become cumbersome [2].

Changing to dimensionless variables x, τ in Eqs. (1.6) and (1.7) and using (1.8) and (1.9), we obtain the expressions for stresses and mass velocities

$$\sigma_r = \rho c_0^2 \left[\frac{k}{1-2a} \frac{c'(\tau)}{x} + \frac{k}{1+a} \frac{c^2(\tau)}{x^4} + c_1(\tau) x^{2(a-1)} + \right.$$
$$\left. + \frac{B}{1-a} \right] \sigma_\theta = a\sigma_r + b, \quad V = c_0 \frac{c(\tau)}{x^2}, \quad B \equiv \frac{b}{\rho c_0^2} \tag{1.10}$$

where $c_1(\tau)$ is a new arbitrary function.

The unknown functions figuring in expressions (1.1), (1.2), (1.5), and (1.10) should be found from boundary conditions which will be formulated for each of the stages of sequential development of the phenomenon, separately for porous and nonporous solid rocks.

2. We will consider the case where the strength and pressure in the cavity are sufficiently great and the rock is nonporous, i.e., where fracture occurs by the formation of cleavage and separation cracks.

This problem for the case where fracture occurs only by separation is solved in [4].

The initial data for which fracture occurs by cleavage can be determined from the solution of the problem for a linearly elastic model with consideration of the condition of fracture [1]

$$\sigma_r - \sigma_\theta = -2\tau_* \tag{2.1}$$

attained in the cavity at instant $\tau = \tau_1$. These data should satisfy the condition [4]

$$P_0 \geqslant 2\tau_* - \sigma_* \tag{2.2}$$

where τ_* is the critical value of tangential stresses, σ_* is the critical value of tensile stresses.

The propagation of the blast waves on fulfilling condition (2.2) occurs in the following stages in the general case.

1. An elastic wave is radiated from the surface of the cavity during time $0 \le \tau \le \tau_1$.

2. The spherical front of cleavage fracture $x = x_2(\tau)$ radiating elastic waves begins to penetrate into the medium from the surface of the cavity at instant $\tau = \tau_1$: fracture continues until instant $\tau = \tau_2$, when the condition of fracture by separation

$$\sigma_\theta = \sigma_* \tag{2.3}$$

is attained on the fracture surface $x = x_2(\tau)$ on the side of the unfractured region.

3. The fracture front bifurcates at instant $\tau = \tau_2$. An elastic wave propagates along the unfractured medium, behind it is the front of fracture by radial cracks ($x = x_1(\tau)$), and behind that is the crushing front ($x = x_2(\tau)$).

4. At instants $\tau = \tau_3$ and $\tau = \tau_4$ the velocity of fronts $x = x_2(\tau)$ and $x = x_1(\tau)$ vanish and fracture stops. The radiation of elastic waves continues until equilibrium is established around the cavity. The equations of the final mathematical problems occurring for the indicated stages of fracture are given below.

The solution of the problem at the first stage is described by formula (1.1), and function $f(\xi)$ is determined by the formula

$$f(\zeta) = \frac{1-\sigma}{2(1-2\sigma)} (p_0 - p_h) \left\{ 1 - \sqrt{2(1-\sigma)} \exp\left[\frac{2\sigma - 1}{1-\sigma} (\zeta + 1) \right] \sin \right.$$
$$\left. \times \left[\frac{\sqrt{1-2\sigma}}{1-\sigma} (\zeta + 1) + \arcsin \frac{1}{2\sqrt{1-\sigma}} \right] \right\}, \quad p_0 = \frac{P_0}{\rho c_0^2} \tag{2.4}$$

Instant τ_1 is determined from condition (2.1) if we substitute into it the values of σ_r and σ_θ in the cavity, using (1.1) and (2.4). The second stage is described by Eqs. (1.1) and (1.5), and the unknown functions $f(\zeta)$, $F_1(\xi_1)$, $F_2(\eta_1)$ are determined from the boundary conditions, which consist of the condition on the cavity and the condition of conjugation at the fracture front, the law of motion of which, $x = x_2(\tau)$, is also to be determined.

The condition on the cavity is obtained from the assumption of an adiabatic quasisteady-state change of the pressure of the blast products upon a change of volume of the cavity [1].

The conditions of conjugation at the front $x = x_2(\tau)$ consist of the fracture condition (2.1), which is fulfilled on the outer side of the front, and of the relationships on the surface of a strong discontinuity.

Taking all this into account, we obtain the equations describing the propagation of blast waves and fracture of the rock in the second stage in the form

$$\frac{f^{\cdot\cdot}(\zeta)}{x_2(\tau)} - 3\left[\frac{f^{\cdot}(\zeta)}{x_2^2(\tau)} + \frac{f(\zeta)}{x_2^3(\tau)}\right] = \frac{2(1-\sigma)}{1-2\sigma} T_*$$

$$F_1^{\cdot}(\xi) - F_2^{\cdot}(\eta) + \frac{F_1(\xi)+F_2(\eta)}{x_2(\tau)} + \frac{4(1-\sigma)}{1+\sigma} T_{*1} x_2^2(\tau)\ln x_2(\tau) = f^{\cdot}(\zeta) + \frac{f(\zeta)}{x_2(\tau)}$$

$$f^{\cdot\cdot}(\zeta) + \frac{2(1-2\sigma)}{1-\sigma}\left[\frac{f^{\cdot}(\zeta)}{x_2(\tau)} + \frac{f(\zeta)}{x_2^2(\tau)}\right] - q^2[F_1^{\cdot\cdot}(\xi) + F_2^{\cdot\cdot}(\eta)] \tag{2.5}$$

$$-4T_{*1}x_2(\tau)\ln x_2(\tau) = x_2^{\cdot}(\tau)\left\{f^{\cdot\cdot}(\zeta) + \frac{f^{\cdot}(\zeta)}{x_2(\tau)} - q\left[F_1^{\cdot\cdot}(\xi) - F_2^{\cdot\cdot}(\eta) + \frac{F_1^{\cdot}(\xi)+F_2^{\cdot}(\eta)}{x_2(\tau)}\right]\right\}$$

$$q^2[F_1^{\cdot\cdot}(\xi^0) + F_2^{\cdot\cdot}(\eta^0)] + p_h = p_0[1 + F_1^{\cdot}(\xi^\circ) - F_2^{\cdot}(\eta^\circ) + F_1(\xi^\circ) + F_2(\eta^\circ)]^{-3\gamma}\quad T_* = \frac{\tau_*}{\rho c_0^2}$$

$$\zeta = \tau - x_2(\tau), \qquad \xi = q\tau - x_2(\tau), \qquad \eta = q\tau + x_2(\tau)$$

$$\xi^\circ = q\tau - 1, \qquad \eta^\circ = q\tau + 1, \qquad T_* = \tau_* / \rho c_0^2$$

Here we ought to supply the quantities ζ, ξ, η with indices in order to distinguish them in Eqs. (1.1), (1.2), and (1.5), but for convenience we will not use indices.

The adiabatic exponent of the blast products γ is taken to be equal to 3 for high pressures and to 1.25 for moderate and low pressures.

The positiveness of a certain expression Λ, which in the case in question amounts to the inequality

$$\Lambda = 2q\tau_{*1}\left\{\frac{F_1^{\cdot\cdot\cdot}(\xi) + F_2^{\cdot\cdot\cdot}(\eta)}{x} + 3\left[\frac{F_1^{\cdot\cdot}(\xi) - F_2^{\cdot\cdot}(\eta)}{x^2} + \frac{F_1^{\cdot}(\xi) + F_2^{\cdot}(\eta)}{x^3}\right]\right\} \geqslant 0 \tag{2.6}$$

is the condition establishing that shear is everywhere plastic behind the front.

In setting up the solution we must watch the sign of Λ, and from the point where for the first time $\Lambda = 0$ we must construct the rarefaction wave behind which shear will be elastic.

We will consider the case where condition (2.6) is fulfilled. From the solution for the first stage we can obtain the condition for the initial data

$$P_0 \geqslant P_h + \frac{2(1-\sigma)}{1-2\sigma}\tau_* \tag{2.7}$$

on fulfillment of which the second stage occurs immediately at the initial instant (first stage is absent) and if not fulfilled the first stage occurs.

System of differential equations (2.5) is integrated to instant $\tau = \tau_2$ during which the condition of fracture by separation (2.3) is attained at the front $x = x_2(\tau)$ on the side of the unfractured material.

The third stage is described by Eqs. (1.1), (1.2), and (1.5), where the unknown functions $f(\zeta)$, $f_1(\xi)$, $f_2(\eta)$, $F_1(\xi_1)$, $F_2(\eta_1)$ are determined from the boundary conditions, which are formed on the cavity and at the fracture fronts, the laws of motion $x = x_1(\tau)$ and $x = x_2(\tau)$ also being subject to determination.

With consideration of (1.1), (1.2), and (1.5) and the indicated boundary conditions, we obtain the following system of functional differential equations for determining the unknown functions of the third stage:

$$q^2[F_1^{\cdot\cdot}(\xi^\circ) + F_2^{\cdot\cdot}(\eta^\circ)] + p_h = p_0[1 + F_1^{\cdot}(\xi^\circ) - F_2^{\cdot}(\eta^\circ) + F_1(\xi^\circ) + F_2(\eta^\circ)]^{-3\gamma}$$

$$f_1^{\cdot}(\xi_1) - f_2^{\cdot}(\eta_1) + \frac{f_1(\xi_1) + f_2(\eta_1)}{x_2(\tau)} = \frac{2T_*}{\lambda^2} x_2(\tau)$$

$$F_1^{\cdot}(\xi_2) - F_2^{\cdot}(\eta_2) + \frac{F_1(\xi_2) + F_2(\eta_2)}{x_2(\tau)} + \frac{4(1-\sigma)}{1+\sigma} T_{*1} x_2^2(\tau)\ln x_2(\tau) = f_1(\xi_1) + f_2(\eta_1) + \frac{1-\sigma}{1+\sigma} p_h x_2^2(\tau)$$

$$2T_* x_2(\tau) - q^2[F_1^{\cdot\cdot}(\xi_2) + F_2^{\cdot\cdot}(\eta_2)] + 4T_{*1}x_2(\tau)\ln x_2(\tau) - p_h x_2(\tau) = x_2^{\cdot}(\tau)\left\{\lambda[f_1^{\cdot}(\xi_1) + f_2(\eta_1)] - q\left[F_1^{\cdot\cdot}(\xi_2) - F_2^{\cdot}(\eta_2) + \frac{F_1^{\cdot}(\xi_2) + F_2^{\cdot}(\eta_2)}{x_2(\tau)}\right]\right\}$$

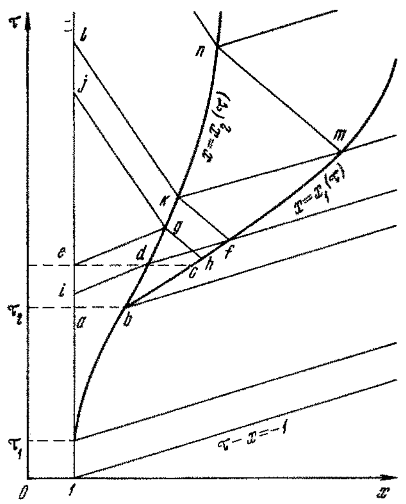

Fig. 1

$$\frac{\sigma}{1-\sigma}\frac{f''(\zeta)}{x_1(\tau)} - \frac{1-2\sigma}{1-\sigma}\left[\frac{f'(\zeta)}{x_1^2(\tau)} + \frac{f(\zeta)}{x_1^3(\tau)}\right] = -\sum\nolimits_* - p_h$$

$$f'(\zeta) + \frac{f(\zeta)}{x_1(\tau)} = f_1(\xi) + f_2(\eta) + \frac{1-\sigma}{1+\sigma}p_h x_1^2(\tau), \qquad \sum\nolimits_* \equiv \frac{\sigma_*}{\rho c_0^2}$$

$$\lambda^2\left[f_1'(\xi) - f_2'(\eta) + \frac{f_1(\xi)+f_2(\eta)}{x_1(\tau)}\right] - f''(\zeta) - \frac{2(1-2\sigma)}{1-\sigma}\left[\frac{f'(\zeta)}{x_1(\tau)}\right.$$

$$\left. + \frac{f(\zeta)}{x_1^2(\tau)}\right] - p_h x_1(\tau) = x_1'(\tau)\left\{\lambda\,[f_1'(\xi) + f_2'(\eta)] - f''(\zeta) - \frac{f'(\zeta)}{x_1(\tau)}\right\} \qquad (2.8)$$

where

$$\zeta = \tau - x_1(\tau), \quad \xi^\circ = q\tau - 1, \quad \xi = \lambda\tau - x_1(\tau)$$
$$\xi_1 = \lambda\tau - x_2(\tau), \quad \xi_2 = q\tau - x_2(\tau), \quad \eta^\circ = q\tau + 1$$
$$\eta = \lambda\tau + x_1(\tau), \quad \eta_1 = \lambda\tau + x_2(\tau), \quad \eta_2 = q\tau + x_2(\tau)$$

The fourth stage arises if on integrating systems (2.5) and (2.8) the velocity of one of the fronts vanishes at some instant ($\tau = \tau_3$). After this instant the fracture condition on the corresponding front [first equation of system (2.5) or second and fifth equations of system (2.8)] must be replaced by an equation corresponding to the condition of stopping of the front

$$x_1(\tau) = \text{const} \quad \text{or} \quad x_2(\tau) = \text{const} \qquad (2.9)$$

Upon exhaustion of the front $x = x_2(\tau)$ its backward motion is impossible, i.e., $x_2{}'(\tau) \geq 0$, and upon exhaustion of front $x = x_1(\tau)$ the condition $x_1{}'(\tau) = 0$ must be preserved up to $\tau = \infty$, if σ_θ at the front $x = x_1(\tau)$ on the side of the unfractured region stays within $0 < \sigma_\theta < \sigma_*$.

If at instant $\tau = \tau_4$ stress σ_θ vanishes, the front $x = x_1(\tau)$ will begin to move into the region of fracture by radial cracks, closing the cracks. From instant τ_4 the solution is described by system (2.8), if in it the right-hand side of the seventh equation is equated to zero and we set $\Sigma_* = 0$.

If the front $x = x_1(\tau)$, performing oscillations, extends to the true boundary between the fractured and unfractured regions, the solution must be continued with consideration of fracture (third stage). In the case where Λ in Eq. (2.6) vanishes at point (x, τ), the solution must be continued with considerations of elastic unloading. The equations for this case are easily obtained from considerations presented in [1]. As specific calculations showed, in all the calculated variants of this problem condition (2.6) is satisfied, and therefore the equations describing elastic unloading will not be presented here.

Let us proceed to a description of the numerical method of solving the problem described above. As was already noted, if condition (2.7) is not satisfied, the solution must be set up by Eqs. (1.1) and (2.4); and if condition (2.7) is satisfied, it must be set up by means of Eqs. (2.5) from the initial instant. In the interval $\tau_1 \leq \tau \leq \tau_2$ the solution of system (2.5) is constructed in the same manner as described in [1, 4], i.e., the asymptotic solution of system (2.5) near the point ($\tau = \tau_1$, $x = 1$) is constructed, system (2.5) is divided into two parts (the first three equations are the first part and the fourth equation is the second part), the Cauchy problem is set up for each of these parts, and then the equations are integrated sequentially. In the first part $F_1(\xi)$ is the known function from the preceding solution (or from the asymptotic solution) and in the second it is the function $F_2(\eta)$.

The solution of system (2.5) is calculated up to instant $\tau = \tau_2$, after which it is necessary to construct the solution of system (2.8). On changing at instant $\tau = \tau_2$ to the construction of the solution of system (2.8) it is necessary to have the asymptotic solution of system (2.8) near the point $\tau = \tau_2$, $x = x_2(\tau)$. We will explain the procedure of continuing the solution by means of a graphic diagram (Fig. 1). In accordance with the scheme described in [1, 4], let the solution be constructed in the region $0 \leq \tau \leq \tau_2$, and by asymptotic formulas be constructed in the small region abcde, where bc and bd are the initial segments of the curves $x = x_1(\tau)$ and $x_2(\tau)$, and ae is a sufficiently small interval. From point d we draw the characteristic $\xi = \lambda\tau - x = \lambda\tau_d - x_2(\tau_d)$. It intersects curve $x = x_1(\tau)$ at some point f. From this point we draw the characteristic $\zeta = \tau - x = \tau_f - x_1(\tau_f)$. Since the solution is known in region abcde, $f_1(\xi)$ will be known in the interval $[\xi_c, \xi_f]$.

In the fifth, sixth, and seventh equations of system (2.8), assuming $f_1(\xi)$ is known, we obtain an individual system for determining the three functions $f(\zeta)$, $f_2(\eta)$, and $x_2(\tau)$. Regarding the latter as functions of $\xi = \tau - x_1(\tau)$ and changing in these equations from differentiation with respect to ξ, η, τ to differentiation with respect to the variable ξ, we reduce the fifth, sixth, and seventh equations of system (2.8) to an individual system of ordinary differential equations for which the Cauchy problem was set up.

The initial data are determined from the asymptotic solution at point c. Solving this problem numerically, we find the segment of the curve of the line $x = x_1(\tau)$, function $f(\xi)$ in the interval $[\xi_c, \xi_f]$, and $f_2(\eta)$ in this case will be known in the interval $[\eta_c, \eta_f]$. We draw through point e the characteristic $\xi_1 = q\tau - x = q\tau_e - 1$; it intersects line $x = x_2(\tau)$ at point g. We then draw the characteristic $\eta = \lambda\tau - x = \lambda\tau_g - x_1(\tau_g)$ from point g; it intersects line $x = x_1(\tau)$ at point h. On segment dg of line $x = x_2(\tau)$ the function $f_2(\eta)$ is known from the solution of the first system; function $F_1(\xi_2)$ will also be known there, since it is known on segment ie [from the asymptotic solution and from the solution of system (2.5)].

From the second, third, and fourth equations of system (2.8) we obtain the second system of equations for determining $F_2(\eta_2)$, $f_1(\xi_1)$, and $x_2(\tau)$. Regarding them as functions of τ and changing from differentiation with respect to ξ_1, ξ_2, η_1, η_2 to differentiation with respect to τ, we reduce these equations to Cauchy's problem for systems ofordinary differential equations with initial data at point d. Solving this (second) system numerically, we find segment dg of line $x = x_2(\tau)$ and functions $F_1(\xi_1)$ and $F_2(\eta_2)$ respectively in intervals $[\xi_{1d}, \xi_{1g}]$ and $[\eta_{1d}, \eta_{1g}]$.

We now draw from point g the characteristic $\eta_1 = q\tau + x = q\tau_g + x_2(\tau_g)$; it intersects line $x = 1$ at point j. In the interval $[\eta_{1a}, \eta_{1j}]$ (on segment aj) function F_2 is known from the initial asymptote and from the solution of the second system. With consideration of this the first equation of system (2.8) on changing to the variable $\eta^* = q\tau + 1$ is reduced to an ordinary differential equation for determing $F_1(\xi_2)$, for which the Cauchy problem with initial data at point e was also set up.

Solving this problem, we determine function F_1 in the interval $[\xi_{1a}, \xi_{1j}]$. Now we draw from point f the characteristic $\eta = \lambda\tau + x = \lambda\tau_f + x_2(\tau_f)$; it intersects line $x = x_2(\tau)$ at point k. From point k we draw the characteristic $\xi_1 = q\tau - x = q\tau_k - x_2(\tau_k)$; it intersects line $x = 1$ at point l. The second system is again solved for interval gk, with $f_2(\eta_1)$ being known in the interval $[\eta_g, \eta_k]$ and $F_1(\xi_2)$ in interval $[\xi_g, \xi_k]$ from the preceding solutions, and Cauchy's data are taken for point g. The first cycle ends on this. The solution is continued further in an analogous order, i.e., drawing through point k the characteristic $\eta = \text{const}$, which intersects line $x = x_1(\tau)$ at point m, we determine the region where the first system must be integrated. Then the second system is integrated on segment kn of curve $x = x_2(\tau)$, after which the equation for $F_1(\xi_2)$, etc. The construction of the solution for late instants is done according to the given scheme, only in Eqs. (2.8) it is necessary to take into account those changes which are described for the fourth stage.

We will proceed to the construction of the asymptotic solutions near instants τ_1 and τ_2. In solving this problem it is necessary to have the asymptotic solution of system (2.5) near point $\tau = 0$, $x = 1$ if condition (2.7) is fulfilled and near point $\tau = \tau_1$, $x = 1$ if condition (2.7) is not fulfilled. It is necessary to have also the asymptotic solution of system (2.8) near point $\tau = \tau_2$, $x = x_2(\tau_2)$.

In the case where condition (2.7) is fulfilled, the solution of system (2.5) near point $\tau = 0$, $x = 1$ is expanded in a Taylor series, i.e., construction of the asymptotic solution amounts to determination of the values of the unknown functions and their derivatives at point (0, 1). From the initial condition in this case $u(x, 0) = 0$ we have

$$f_0 = f_0^{\cdot} = 0, \quad f_0^{\cdot\cdot} = \frac{2(1-\sigma)}{1-2\sigma} T_* \tag{2.10}$$

i.e., at the initial instant the stresses and mass velocities in the unfractured region depend only on the parameters of the medium. Here and henceforth the indices 0 are the values of the functions and their derivatives at points corresponding to the start of fracture (points (0, 1), $(\tau_1, 1)$, $(\tau_2, x_2(\tau_2))$).

From system (2.5) we obtain for the initial velocity of the front

$$x_{20}^{\cdot} = \left[\frac{(1-2\sigma)(1+\sigma)}{1-\sigma} \frac{2(1-\sigma)T_* - p_0 - p_h}{2(1+\sigma)T_* + (1-2\sigma)(4T_{*1} - 3p_0 - 3p_h)}\right]^{1/2} \tag{2.11}$$

Hence follows in particular that $x_{20}^{\cdot} \to q$ as $p_0 - p_h \to \infty$, i.e., the propagation velocity of the fracture front is limited, which also follows from the condition of thermodynamic correctness of the problem obtained in [1].

For constructing the asymptotic solution we do not need all values of the functions and their derivatives but only the following combinations figuring in Eqs. (1.5):

$$F_1^{\cdot} - F_2^{\cdot} + F_1 + F_2, \quad F_1^{\cdot\cdot} + F_2^{\cdot\cdot}, \quad F_1^{\cdot\cdot} - F_2^{\cdot\cdot} + F_1^{\cdot} + F_2^{\cdot} \tag{2.12}$$

By virtue of the existing arbitrary rule we assume

$$F_{10} = F_{20} = F_{10}^{\cdot} = F_{20}^{\cdot} = 0 \tag{2.13}$$

From system (2.5) we easily find for point $\tau = 0$, $x = 1$

$$\begin{gathered}
F_{n0}^{\cdot\cdot} = \frac{p_0 - p_h}{2q^2} - (-1)^n a_1 \\
F_{n0}^{\cdot\cdot\cdot} = -1.5\gamma q^2 p_0 a_1 - (-1)^n \frac{x_{20}^{\cdot\cdot} a_3 - a_2}{q^2 - x_{20}^{\cdot 2}} \quad (n = 1, 2) \\
f_0^{\cdot\cdot\cdot} = \frac{2(1-\sigma)}{1-2\sigma} \frac{4x_{20}^{\cdot} - 3}{1 - x_{20}^{\cdot}} T_* \\
x_{20}^{\cdot\cdot} = \frac{(1-2\sigma)[q^2(a_4 - 2a_2 x_{20}^{\cdot}) - a_4 x_{20}^{\cdot 2}]}{[2(1-\sigma)T_* - q(1-2\sigma)a_1](q^2 - x_{20}^{\cdot 2}) - 2qa_3(1-2\sigma)x_{20}^{\cdot}}
\end{gathered} \tag{2.14}$$

Here

$$\begin{gathered}
a_1 = \frac{1}{2q}\left\{\frac{2(1-\sigma)}{1-2\sigma} T_* - \left[\frac{2(1-2\sigma)}{1-2\sigma} - p_0 - p_h\right]^{1/2}\left[\frac{4(1-\sigma)}{1+\sigma} T_{*1} - \frac{3(1-\sigma)}{1+\sigma}(p_0 - p_h) + \frac{2(1-\sigma)}{1-2\sigma}\right]^{1/2}\right\} \\
a_2 = \frac{6p_0}{q}\gamma a_1 x_{20} - (q - x_{20})^2 F_{10}^{\cdot\cdot} + (q + x_{20}^{\cdot})^2 F_{20}^{\cdot\cdot} + \frac{12(1-\sigma)}{1+\sigma} T_* x_{20}^{\cdot 2} - (1 - x_{20}^{\cdot})^2 (f_0^{\cdot\cdot\cdot} - f_0^{\cdot\cdot}) \\
a_3 = \frac{1}{q^2}(p_0 - p_h) - \frac{2(1-\sigma)}{1-2\sigma} T_* - \frac{4(1-\sigma)}{1+\sigma} T_{*1} \\
a_4 = (1 - x_{20}^{\cdot})(f_0^{\cdot\cdot\cdot} + T_*) + 3\gamma p_0 q a_1 + 4T_{*1} x_{20} - q - x_{20}^{\cdot}\Big\{(1 - x_{20}^{\cdot})(f_0^{\cdot\cdot\cdot} \\
+ f_0^{\cdot\cdot}) - \frac{3\gamma}{q} p_0 x_{20} - q[(q - x_{20}^{\cdot})F_{10}^{\cdot\cdot} + (q + x_{20}^{\cdot})F_{20}^{\cdot\cdot}]\Big\}
\end{gathered} \tag{2.15}$$

There is no need to determine higher order derivatives, since the problem must be solved numerically, and the required accuracy can be attained by selecting a sufficiently small initial segment.

In the case where condition (2.7) is not fulfilled, the asymptotic solution of system (2.5) near point $(\tau_1, 1)$ contains a singularity and is constructed as in [4]. From the solution of the first stage f_0 and $f_0^{\cdot}$ are known.

With consideration of this and the arbitrary rule (2.12), from system (2.5) we can easily determine $f_0^{\cdot\cdot}$, $f_0^{\cdot\cdot\cdot}$, F_{10}, $F_{10}^{\cdot}$, F_{20}, $F_{20}^{\cdot}$ for point $(\tau_1, 1)$, and relation $x_{20}^{\cdot} = 0$ follows from the continuity of σ_r on characteristic $\zeta = \tau_1 - 1$ and from the condition of continuity of impulse, since $[du/dr] \neq 0$.

Functions $x_2^{\cdot\cdot}(\tau)$, $F_1^{\cdot\cdot}(\xi)$, $F_2^{\cdot\cdot}(\eta)$, and $f^{(IV)}(\zeta)$ have singularities of order

$$x_2^{\cdot\cdot} \sim (\tau - \tau_1)^{-1/2}, \quad F_1^{\cdot\cdot} \sim (\xi - \xi_0)^{-1/2}, \quad F_2^{\cdot\cdot} \sim (\eta - \eta_0)^{-1/2}, \quad f^{(IV)} \sim (\zeta - \zeta_0)^{-1/2} \tag{2.16}$$

With consideration of the aforesaid, we obtain the asymptotic solution of system (2.5) near point $\tau = \tau_1$, $x = 1$ in the form

$$\begin{gathered}
x_2(\tau) = 1 + b_0(\tau - \tau_1)^{3/2} + \ldots \\
F_1^{\cdot}(\xi_1) = b_1 + b_2 q(\tau - \tau_1) - b_0 b_2 (\tau - \tau_1)^{3/2} + 0.5 b_3 q(\tau - \tau_1)^2 + q(0.53 q^2 b_4 \\
- b_0 b_3)(\tau - \tau_1)^{5/2} - 0.5 b_0 (2.83 q^2 b_4 - b_0 - b_3)(\tau - \tau_1)^3 + \ldots \\
F_2(\eta_1) = b_2 q(\tau - \tau_1) + b_0[b_2 - 0.4 q b_5 (q - b_0)](\tau - \tau_1)^{3/2} - [0.5q^2 - 0.45 b_0 b_5 (q \\
- b_0)](\tau - \tau_1)^2 - q[0.53 q^2 b_4 - b_0 b_3 - 0.33 q b_5 (q - b_0)](\tau - \tau_1)^{5/2} \\
- [1.42 b_0 b_4 q^2 - 0.5 b_0^2 b_3 - 0.16 b_5 (q - b_0)](\tau - \tau_1)^3 + \ldots \\
f(\zeta) = f_0 + f_0^{\cdot}(\tau - \tau_1) - b_0 f_0 (\tau - \tau_1)^{3/2} + 0.5 f_0^{\cdot\cdot}(\tau - \tau_1)^2 + b_0(0.4 + 0.6 f_0^{\cdot\cdot}) \\
(\tau - \tau_1)^{5/2} + (0.16 f_0^{\cdot\cdot\cdot} + 0.5 b_0^2)(\tau - \tau_1)^3 + \ldots \\
b_0 = \left[\frac{8}{3} \frac{(b_5 q^3 - f_0^{\cdot\cdot\cdot})(1-\sigma) - 4qb_2(1-2\sigma)}{(2b_3 - f_0^{\cdot\cdot\cdot})(1+\sigma) - 4T_{*1}(1-\sigma)}\right]^{1/2} \\
b_1 = f_0 + f_0^{\cdot}, \quad b_2 = \frac{1}{2q}(f_0^{\cdot} + f_0^{\cdot\cdot}), \quad b_3 = \frac{1}{2q^2}[(p_0 + b_1)^{-3\gamma} - p_h]
\end{gathered} \tag{2.17}$$

$$b_4 = 0{,}75 b_3 - \frac{3(1-\sigma)}{4q^2}\left(\frac{2}{1+\sigma}T_{*1} + \frac{1}{1-2\sigma}T_*\right), \qquad f_0^{\cdot\cdot\cdot} = -3\,(f_0^{\cdot} + f_0^{\cdot\cdot})$$

$$b_5 = \frac{6\gamma}{q^2} b_0 p_0 (1 + b_1), \qquad f_0^{\cdot\cdot} = \frac{2(1-\sigma)}{1-2\sigma}T_* - 3b_1 \tag{2.18}$$

The solution of system (2.8) near point $\tau = \tau_2$, $x = x_2(\tau_2)$ is expanded in a Taylor series, i.e., the problem amounts to determining the values of the unknown functions and their derivatives at this point. On the characteristics $\xi = \tau - x = \tau_2 - x_2(\tau_2)$ and $\xi_2 = q\tau - x = q\tau_2 - x_2(\tau_2)$ the displacement and stresses are continuous, i.e., $f_0, f_0^{\cdot}, f_0^{\cdot\cdot}, F_{10}, F_{10}^{\cdot}, F_{10}^{\cdot\cdot}, F_{20}, F_{20}^{\cdot}, F_{20}^{\cdot\cdot}, x_{10} = x_{20}$ are known from the preceding solution.

From system (2.8) we easily determine the quantities

$$f_{10} = f_0^{\cdot} + \frac{f_0}{x_{20}} - \frac{1-\sigma}{1+\sigma} p_h x_{20}^2, \qquad f_{20} = 0$$

$$f_{n0} = \frac{1}{2\lambda}\left[d_0 - \sqrt{d_1 d_2}\right] - (-1)^n \left(\frac{x_{20}}{\lambda^2} T_* - \frac{f_{10}}{x_{20}}\right) \qquad (n = 1, 2) \tag{2.19}$$

$$x_{20}^{\cdot} = \left(\frac{d_2}{d_1}\right)^{1/2}, \quad x_{10}^{\cdot} = \frac{d_3}{(f_{10}^{\cdot} + f_{20}^{\cdot} - f_0^{\cdot\cdot\cdot})\, x_{20} - f_0^{\cdot}}$$

where

$$d_0 = \frac{q}{x_{20}}\left[(F_{10}^{\cdot\cdot} - F_{20}^{\cdot\cdot})\, x_{20} + F_{10}^{\cdot} + F_{20}^{\cdot}\right]$$

$$d_1 = F_{10}^{\cdot\cdot} + F_{20}^{\cdot\cdot} + \frac{1}{x_{20}}(F_{10}^{\cdot} - F_{20}^{\cdot}) - \frac{1}{x_{20}^2}(F_{10} + F_{20}) +$$

$$+ \frac{4(1-\sigma)}{1+\sigma} T_{*1} x_{20} (1 + 2\ln x_{20}) - \frac{2x_{20}}{\lambda} T_* - \frac{f_{20}}{x_{20}} - \frac{2(1-\sigma)}{1+\sigma} p_h x_{20} \tag{2.20}$$

$$d_2 = q^2 (F_{10}^{\cdot\cdot} + F_{20}^{\cdot\cdot}) + 2x_{20} T_* - 4x_{20} T_{*1} \ln x_{20} + p_0 x_{20}$$

$$d_3 = \lambda^2 \left(f_{10}^{\cdot} - f_{20}^{\cdot} + \frac{f_{10}}{x_{20}}\right) - f_0^{\cdot\cdot} - \frac{2(1-2\sigma)}{1-\sigma}\left(\frac{f_0^{\cdot}}{x_{20}} + \frac{f_0}{x_{20}^2}\right) - p_h x_{20}$$

The expressions of higher order derivatives can be determined easily by differentiating Eqs. (2.8) and arranging τ to τ_2.

On the "Strela-4" computer the algorithm described above was used to calculate the problem of the effect of a blast in various media with initial pressures in the cavity $P_0 = 2 \cdot 10^3 - 10^5$ atm up to the instant of formation of a region of radial cracks. Some auxiliary calculations were made on the "Nairi" computer.

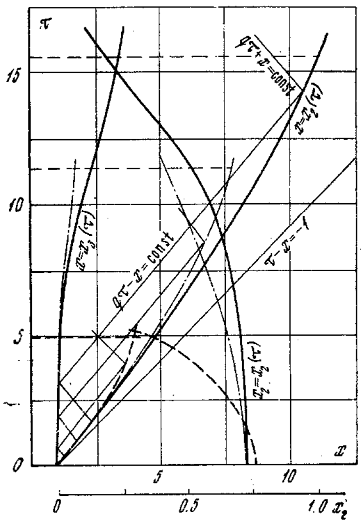

Fig. 2

The results of the calculations for the case where condition (2.7) is fulfilled are shown in plane $x\tau$ in Fig. 2. The law of expansion of the cavity $x = x_3(\tau)$, law of motion of the fracture front $x = x_2(\tau)$, and change of velocity of the front $x_2^{\cdot}(\tau)$ with time in the interval $0 \leq \tau \leq \tau_2$ are shown.

The results pertain to clay shale, solid lines ($\sigma = 0.26$, $E = 1.9 \cdot 10^5$ kg/cm^2, $\sigma_* = 38$ kg/cm^2, $\tau_* = 250$ kg/cm^2, $\tau_{*1} = 100$ kg/cm^2); to limestone, dot-dash lines ($\sigma = 0.25$, $E = 7 \cdot 10^5$ kg/cm^2, $\sigma_* = 25.5$ kg/cm^2, $\tau_* = 400$ kg/cm^2, $\tau_{*1} = 150$ kg/cm^2); to granite, dashed line ($\sigma = 0.3$, $E = 2.22 \cdot 10^5$ kg/cm^2, $\sigma_* = 45$ kg/cm^2, $\tau_* = 750$ kg/cm^2, $\tau_{*1} = 400$ kg/cm^2) [3] for a blast with parameters $P_0 = 10^4$ atm, $P_h = 10$ atm.

Figure 3 shows similar data for the case where condition (2.7) is not fulfilled for a blast in granite with parameters $P_0 = 2000$ atm, $P_h = 300$ atm (dashed lines) and $P_0 = 2500$ atm, $P_h = 300$ atm (solid lines). As in Fig. 2, the calculation was performed only for the interval $0 \leq \tau \leq \tau_2$ except that here $\tau_1 > 0$. The graph of the expansion of the cavity is not shown in Fig. 3, since this expansion is negligibly small in the given case.

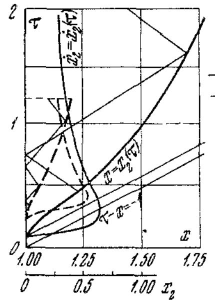

Fig. 3

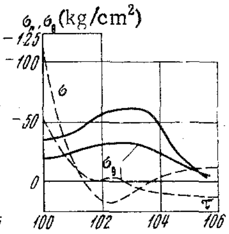

Fig. 4

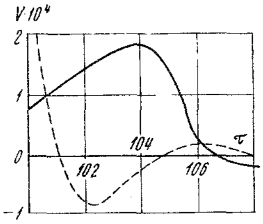

Fig. 5

Here, as in the case of the formation of only a region of fracture cracks [4], the relations $x_2^{\cdot}(\tau_1) = 0, x_2^{\cdot\cdot}(\tau_1) = \infty$ hold true, i.e., the fracture front begins to move from the surface of the cavity with a zero velocity and in a short time picks up maximum velocity (in this case, of course, the limitation on the velocity of the fracture front obtained from the requirements of thermodynamic correctness of the problem formulated and uniqueness of its solution [1] is realized), after which $x_2^{\cdot\cdot} < 0$ – the velocity of the front decreases slowly.

As we see from Fig. 2 and from the results of the calculations of many other variants, with sufficiently high initial pressures the velocity of the fracture front at first decreases very slowly in comparison with late instants. After the velocity of the front begins to decrease intensely, condition (2.3) is attained at the fracture front before the front velocity vanishes.

Figures 4 and 5 show the laws of change of stresses and mass velocities in time at distances $x = 100$ for a blast with parameters $P_0 = 10^4$ atm, $P_h = 10$ atm in granite, and also the corresponding data for the solution of the problem in a purely elastic setup.

We see that the solution constructed differs considerably from the solution of the problem in a purely elastic setup, both with respect to the form of the elastic wave and to its amplitude and duration, i.e., despite the fact that the region of fracture by cleavage ($r \sim 5r_0$) is small in comparison with the distance considered ($r = 100\,r_0$), the effect of processes in the fractured region on the character of the elastic waves radiated is substantial. True, in the case in question the effects of a decrease of amplitude and increase of duration of the elastic wave in comparison with the case of the purely elastic solution are not as great as in the cases considered in Section 3 of this paper and in [5].

Figure 6 shows the graphs of the change of mass velocities in time at various distances from the blast center in granite for parameters $P_0 = 10^4$ atm, $P_h = 10$ atm.

Figure 7 shows the profiles of the radial and hoop (dot-dash lines) stresses in the near zone for a blast with parameters $P_0 = 10^4$ atm, $P_h = 10$ atm in granite for different instants noted in the figure.

The first jump of stresses corresponds to the arrival of the leading edge of the radiated elastic wave and the second to the passage of the fracture front.

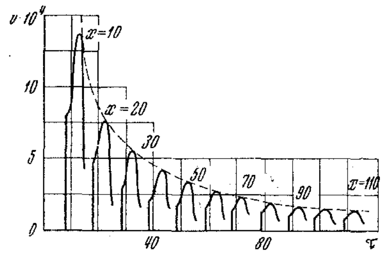

Fig. 6

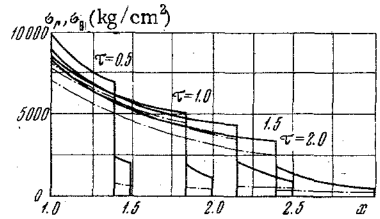

Fig. 7

We note that it follows from an analysis of the results of calculations for different variants in which τ_* and τ_{*1} were varied that, other conditions being equal, the dimensions of the region of fractured rock increase with decrease of the difference between τ_* and τ_{*1}. If $\tau_{*1} \to \tau_*$, the velocity of the fracture front tends to a constant value equal to c_2. This case with consideration of the condition $\sigma_* = 0$ corresponds to jointed rocks. Some exact solutions for this special case were obtained in [6].

3. We will consider the problem of the effect of a blast in porous brittle rocks. The porous material is fractured when the stresses exceed the critical values of the tensile (σ_*), compressive (σ_{**}), and shear ($\tau_* = 0.5\sigma_{**}$) stresses and hydrostatic stress (σ_{*0}).

The fracture conditions for porous rocks can be written in the form

$$\sigma_\theta = \sigma_*, \qquad \sigma_r + \alpha\sigma_\theta = -\sigma_{**} \tag{3.1}$$

For $\alpha = 0$ we have fracture when the radial stress reaches the critical compressive stress. The condition $\alpha = -1$ corresponds to the case of fracture when the tangential stresses reach the critical value. For $\alpha = 2$ the material fractures when the stresses reach the critical value of hydrostatic stress.

If the initial pressure in the cavity is high, a supersonic spherical crushing shock front will pass through the rock at the initial instant. The velocity of the crushing front will decrease with time, and at the instant when the velocity becomes equal to the velocity of sound in the unfractured material, the front will begin to radiate an elastic wave into the unfractured material. If tensile stresses reaching the critical value σ_* occur in the medium not fractured by crushing, a front of fracture by separation cracks will pass through the medium.

At this instant the fracture front bifurcates – the fracture front brought about by separation cracks, which fractures the material by radial cracks, proceeds ahead, and behind it comes the crushing front, which crushes the material into small blocks. Radiation of elastic waves into the unfractured material will continue, and the fracture fronts will become exhausted with time, i.e., their velocities will vanish. The radiation of elastic waves in the vicinity of the blast cavity will continue until equilibrium occurs there.

The unknown functions $f(\zeta), f_1(\xi), f_2(\eta)$, $c(\tau)$, $c_1(\tau)$ in expressions (1.1), (1.2), and (1.10), the laws of propagation of the fracture fronts, and the law of expansion of the cavity are determined from the boundary conditions. We will derive below the final systems of equations for individual successive stages of fracture.

First Stage. A supersonic crushing shock front propagates through the undisturbed medium. Here the unknown functions are the law of expansion of the cavity $r = r_3(t)$, law of propagation of the crushing front $r = r_2(t)$, and $c(\tau)$ and $c_1(\tau)$. These functions are determined from the boundary conditions

$$\begin{gathered} r_3^{\cdot} = V|_{r=r_3}, \qquad \sigma_r|_{r=r_3} = -P_0 (r_3/r_0)^{-3\gamma} \\ V|_{r=r_2} = \frac{k-1}{k} r_2^{\cdot}, \quad \sigma_r|_{r=r_2} = -P_h - \rho V r_2^{\cdot} \end{gathered} \tag{3.2}$$

Changing in (3.2) to dimensionless coordinates x, τ and taking into account the expressions for σ_r and V from (1.10), we obtain a system of ordinary differential equations

$$\begin{gathered} x_3^2(\tau)\, x_3^{\cdot}(\tau) = c(\tau), \qquad x_2^2(\tau)\, x_2^{\cdot}(\tau) = \frac{k}{k-1} c(\tau) \\ \frac{k}{1-2a} c^{\cdot}(\tau) + \frac{k}{1+a} c^2(\tau)\, x_3^{-3}(\tau) + c_1(\tau)\, x_3^{2a-1}(\tau) = p_0 x_3^{-3\gamma-1}(\tau) - \frac{B}{1-a} x_3(\tau) \\ \frac{k}{1-2a} c^{\cdot}(\tau) + \frac{k}{1+a} c^2(\tau)\, x_2^{-3}(\tau) + c_1(\tau)\, x_2^{2a-1}(\tau) = -\left(p_h + \frac{B}{1-a}\right) x_2(\tau) - \frac{k-1}{k} x_2^{\cdot 2}(\tau)\, x_2(\tau) \end{gathered} \tag{3.3}$$

with initial conditions

$$x_3(0) = x_2(0) = 1, \qquad c(0) = \left(\frac{k-1}{k}(p_0 - p_h)\right)^{1/2} \tag{3.4}$$

The initial velocities of the expansion of the cavity and of the crushing front are determined respectively by the formulas

$$x_3^{\cdot}(0) = \left(\frac{k-1}{k}(p_0 - p_h)\right)^{1/2}, \quad x_2^{\cdot}(0) = \left(\frac{k}{k-1}(p_0 - p_h)\right)^{1/2} \tag{3.5}$$

The solution of system (3.3) is valid in region $x_2^{\cdot}(\tau) \geq 1$. At instant $\tau = \tau_1$, where the velocity of the crushing front becomes equal to the propagation velocity of shock waves in the unfractured medium c_0, the second stage begins, and the solution must be continued with consideration of the radiation of elastic waves.

Second Stage. An elastic wave, which is the crushing front, propagates through the unfractured material. Here the unknown functions are $x_3(\tau)$, $x_2(\tau)$, $c(\tau)$, $c_1(\tau)$, $f(\zeta)$. These functions are found from the following boundary conditions:

$$r_3^{\cdot} = V|_{r=r_3}, \quad \sigma_r|_{r=r_3} = -P_0(r_3/r_0)^{-3\gamma}, \quad V_2 - V_1 = (\rho_2 - \rho_1)\rho_2^{-1} r_2^{\cdot}$$
$$\sigma_{r2} - \sigma_{r1} = \rho_1 r_2^{\cdot}(V_1 - V_2), \qquad \sigma_{r1} + \alpha\sigma_{\theta 1} = -\sigma_{**} \tag{3.6}$$

The indices 1 and 2 denote quantities ahead of and behind the fracture front, respectively. We substitute the expressions of σ_r, σ_θ, and V from (1.1) and (1.10) into (3.6) in dimensionless coordinates x, τ with consideration of (1.9) and formula

$$\rho_1 = \rho\left(1 - \frac{\partial u_1}{\partial r} - \frac{2u_1}{r}\right) \tag{3.7}$$

As a result we obtain the following system of differential equations for determining the five unknown functions

$$\frac{k}{1-2a}c^{\cdot}(\tau) + \frac{k}{1+a}c^2(\tau)x_2^{-3}(\tau) + c_1(\tau)x_3^{2a-1}(\tau) = -p_0 x_3^{-3\gamma-1}(\tau) - \frac{B}{1-a}x_3(\tau)$$
$$x_3^2(\tau)\,x_3^{\cdot}(\tau) = c(\tau)$$
$$c(\tau) - f^{\cdot\cdot}(\zeta_2) - \frac{f^{\cdot}(\zeta_2)}{x_2(\tau)} = \frac{x_2(\tau)}{k}[(k-1)x_2(\tau) - f^{\cdot\cdot}(\zeta_2)]$$
$$\frac{k}{1-2a}c^{\cdot}(\tau) + \frac{k}{1+a}c^2(\tau)x^{-3}(\tau) + c_1(\tau)x_2^{2a-1}(\tau) + \frac{B}{1-a}x_2(\tau) + f^{\cdot\cdot}(\zeta_2)$$
$$+ \frac{2(1-2\sigma)}{1-\sigma}\left[\frac{f^{\cdot}(\zeta_2)}{x_2(\tau)} + \frac{f(\zeta_2)}{x_2^2(\tau)}\right] + p_h x_2(\tau) = -x_2^{\cdot 2}(\tau)[(k-1)x_2^{\prime}(\tau) - f^{\cdot\cdot}(\zeta_2)] \tag{3.8}$$
$$\frac{1-\sigma(1-\alpha)}{1-\sigma}f^{\cdot\cdot}(\zeta_2) + \frac{(1-2\sigma)(2-\alpha)}{1-\sigma}\left[\frac{f^{\cdot}(\zeta_2)}{x_2(\tau)} + \frac{f(\zeta_2)}{x_2(\tau)}\right]$$
$$= -[\Sigma_{**} + (1+\alpha)p_h]x_2(\tau), \quad \zeta_2 = \tau - x_2(\tau), \quad \Sigma_{**} = \frac{\sigma_{**}}{\rho c_0^2}$$

If the condition

$$p_0 \geqslant \frac{k-1}{k} + \frac{p_h}{k} + \frac{k-1}{k}\Sigma_{**} \tag{3.9}$$

is fulfilled, the initial data for system (3.8) are taken from the solution of system (3.3) at point $\tau = \tau_1$, $x = x_2(\tau)$. If condition (3.9) is not fulfilled, i.e., the second stage occurs immediately at the initial instant (the crushing front will be subsonic), the initial condition will be

$$f(-1) = f^{\cdot}(-1) = 0, \quad x_2(0) = x_3(0) = 1$$
$$c(0) = \Sigma_{**} - p_h + \left[(p_0 - \Sigma_{**})\left(\frac{k-1}{k} - \frac{\Sigma_{**} - p_h}{k}\right)\right]^{1/2} \tag{3.10}$$

The solution of system (3.8) is constructed either before the instant when the hoop stresses on the outer side of the crushing front become critical, or before the instant when the crushing front is exhausted.

If at first the second of these possibilities is realized, then since the backward movement of the crushing front is impossible, this front must hereafter be replaced by a contact discontinuity. In this case we must eliminate the fifth equation (fracture condition) from system (3.8) and replace it by the equation

$$x_2(\tau) = \text{const} \tag{3.11}$$

The solution thus obtained will describe the propagation of elastic waves in the unfractured region and plastic flow in the crushing region in the absence of new fractures of the medium. This solution must be continued until the instant when a zone of radial cracks occurs at the contact discontinuity on the side of the unfractured material. Such continuation of the solution for rocks is improbable, since for them $\sigma_* \ll \sigma_{**}$. It can occur in materials for which σ_* and σ_{**} are commensurable (for example, Plexiglas [7]). We will consider the first case, when the zone of radial cracks occurs for $x_2^{\cdot}(\tau) > 0$. The corresponding instant τ_2 is determined from the equation

$$\frac{\sigma}{1-\sigma}f^{\cdot\cdot}(\zeta_*) - \frac{1-2\sigma}{1-\sigma}\left[\frac{f^{\cdot}(\zeta_*)}{x_2(\tau_2)} + \frac{f(\zeta_*)}{x_2^2(\tau_2)}\right] = -(p_h + \Sigma_*)x_2(\tau_2) \tag{3.12}$$

$$\xi_* = \tau_2 - x_2(\tau_2), \quad \Sigma_* = \frac{\sigma_*}{\rho c_0^2}$$

The third stage arises at this instant.

Third Stage. An elastic wave separated from the zone of radial cracks by front $x = x_1(\tau)$ propagates through the unfractured material: behind the zone of radial cracks is the crushing zone, the boundary between them being the crushing front $x = x_2(\tau)$. Here $x_3(\tau)$, $x_2(\tau)$, $x_1(\tau)$, $c_1(\tau)$, $f(\zeta)$, $f_1(\xi)$, $f_2(\eta)$ are the unknown functions which should be found from the following boundary conditions:

condition on the cavity

$$r_3^{\cdot} = V|_{r=r_3}, \qquad \sigma_r = -P_v(r_3/r_0)^{-3\gamma} \tag{3.13}$$

condition on the crushing front

$$V_2 - V_1 = \frac{\rho_2 - \rho_1}{\rho_2} r_2^{\cdot}, \quad \sigma_{r2} - \sigma_{r1} = \rho_1 r_2^{\cdot}(V_1 - V_2), \quad \sigma_{r1} = \sigma_{**} \tag{3.14}$$

condition on the front of fracture by radial cracks

$$u_1 = u_2, \quad \sigma_{r2} - \sigma_{r1} = \rho r_1^{\cdot}(V_1 - V_2), \quad \sigma_{\theta 1} = \sigma_* \tag{3.15}$$

Writing (3.13)-(3.15) in dimensionless coordinates x, τ and substituting the corresponding expressions from Eqs. (1.1), (1.2), and (1.10), we obtain the following functional differential equations for determining the unknown functions:

$$\begin{gathered}
\frac{k}{1-2a} c^{\cdot}(\tau) + \frac{k}{1+a} c^2(\tau) x_3^{-3}(\tau) + c_1(\tau) x_3^{2a-1}(\tau) + \frac{B}{1-a} x_3(\tau) = -p_0 x_3^{-3\gamma-1}(\tau) \\
x_3^2(\tau) x_3^{\cdot}(\tau) = c(\tau), \qquad c(\tau) - \lambda x_2(\tau)[f_1^{\cdot}(\xi_2) + f_2^{\cdot}(\eta_2)] \\
= \frac{x_2^{\cdot}(\tau)}{k}\Big\{(k-1)x_2^2(\tau) - [f_1^{\cdot}(\xi_2) - f_2^{\cdot}(\eta_1)]x_2(\tau) - f_1(\xi_2) \\
- f_2(\eta_2) - \frac{3(1-\sigma)}{1+\sigma} p_h x_2^2(\tau)\Big\} \\
\frac{k}{1-2a} c^{\cdot}(\tau) + \frac{k}{1+a} c^2(\tau) x_2^{-3}(\tau) + c_1(\tau) x_2^{2a-1}(\tau) + \left(\Sigma_{**} + \frac{B}{1-a}\right) x_2(\tau) \\
= x_2^{\cdot}(\tau)\Big\{\lambda[f_1^{\cdot}(\xi_2) + f_2^{\cdot}(\eta_2)] - \frac{c(\tau)}{x_2(\tau)}\Big\}\Big[1 + \frac{f_1^{\cdot}(\xi_2) - f_2^{\cdot}(\eta_2)}{x_2(\tau)} - \frac{f_1(\xi_2) + f_2(\eta_2)}{x_2^2(\tau)} - \frac{3(1-\sigma)}{1+\sigma} p_h\Big] \\
f_1^{\cdot}(\xi_2) - f_2^{\cdot}(\eta_2) + \frac{f_1(\xi_2) + f_2(\eta_2)}{x_2(\tau)} = \frac{\Sigma_{**}}{\lambda^2} x_2(\tau) \\
\lambda^2\Big[f_1^{\cdot}(\xi_1) - f_2^{\cdot}(\eta_1) + \frac{f_1(\xi_1) + f_2(\eta_1)}{x_1(\tau)}\Big] - f^{\cdot\cdot}(\zeta_1) - \frac{2(1-2\sigma)}{1-\sigma}\Big[\frac{f^{\cdot}(\zeta_1)}{x_1(\tau)} + \frac{f(\zeta_1)}{x_1^2(\tau)}\Big] \\
- p_h x_1(\tau) = x_1^{\cdot}(\tau)\Big\{\lambda[f_1^{\cdot}(\xi_1) + f_2^{\cdot}(\eta_1)] - f^{\cdot\cdot}(\zeta_1) - \frac{f^{\cdot}(\zeta_1)}{x_1(\tau)}\Big\} \\
\frac{\sigma}{1-\sigma} f^{\cdot\cdot}(\zeta_1) - \frac{1-2\sigma}{1-\sigma}\Big[\frac{f^{\cdot}(\zeta_1)}{x_1(\tau)} + \frac{f(\zeta_1)}{x_1^2(\tau)}\Big] = -(\Sigma_* + p_h) x_1(\tau)
\end{gathered} \tag{3.16}$$

where

$$\begin{gathered}
\zeta_1 = \tau - x_1(\tau), \quad \xi_1 = \lambda\tau - x_1(\tau), \quad \xi_2 = \lambda\tau - x_2(\tau) \\
\eta_1 = \lambda\tau + x_1(\tau), \quad \eta_2 = \lambda\tau + x_2(\tau)
\end{gathered} \tag{3.17}$$

Equations (3.16) describing fractures and propagation of blast waves at the third stage are integrated up to the instant when the velocity of one of the fracture fronts vanishes ($x_1^{\cdot} = 0$ or $x_2^{\cdot} = 0$), and the fourth stage begins. It is more probable that the crushing front is exhausted first.

Fourth Stage. An elastic wave propagates through the unfractured material and behind it the wave of radial cracks, the boundary between the crushing zone and the radial crack zone being a contact discontinuity. Plastic flow continues and new crushing of the medium does not occur.

After instant $\tau = \tau_3 (x_2^{\cdot}(\tau_3) = 0)$ there can be backward movement of the front $x = x_2(\tau)$, and the fifth equation of system (3.16) must be replaced by Eq. (3.11).

The equations of the fourth stage are integrated up to instant $\tau = \tau_4$, when the front of fracture by separation ($x_1^{\cdot}(\tau_4) = 0$) is exhausted and the fifth stage begins.

Fifth Stage. An elastic wave propagates through the unfractured material, the front of fracture by radial cracks has also stopped, the boundaries between the three zones are contact discontinuities, plastic flow continues, and no new fractures of the medium occur.

The system of equations of this stage is obtained from the system of equations of the fourth stage, if the eighth equation (condition of fracture by separation) is replaced by the equation

$$x_1(\tau) = \text{const} \tag{3.18}$$

If the hoop stress on the side of the unfractured material, dropping to value σ_*, approaches its positive asymptotic value, the solution of the fifth stage can be continued to $\tau = \infty$, and if at instant $\tau = \tau_5$ it vanishes and changes sign, the sixth stage occurs and the solution must be continued differently.

Sixth Stage. Everything occurs as in the fifth stage, only the front of fracture by radial cracks performs backward movement into the fractured zones, closing the cracks.

The system of equations in this stage is obtained from the system of equations of the fourth stage if we equate the left-hand side of the seventh equation of system (3.16) to zero and set there $\Sigma_* = 0$. The solution can be continued to $\tau = \infty$ if new fractures of the medium do not occur. If they do occur, i.e., at instant $\tau = \tau_6$ the boundary $x = x_1(\tau)$ extends to the true boundary of fracture, it is necessary to integrate the equations of the fourth stage, then the fifth, etc.

Thus, systems of equations are obtained for all stages. The sequence of certain stages can vary, depending on the properties of the rock, initial pressure in the rock, and properties of the explosive.

Generally speaking, the described problem can be solved only numerically with the use of a digital computer. For the first and second stages we have a system of ordinary differential equations (3.3), (3.8) for which the Cauchy problem is set up. For the third, fourth, and sixth stages we have a system of functional differential equations. By means of the methods described above these systems are reduced to a sequence of Cauchy problems for certain systems of ordinary differential equations. For the fifth stage a system of differential equations with a divergent argument is obtained, the solution of which is constructed analogously.

To continue the solution at the third stage, it is necessary to have the asymptotic solution of system (3.16) near point $\tau = \tau_2$, $x = x_2(\tau_2)$. Here the asymptotic solution does not have a singularity, and the unknown functions are expanded in Taylor series. In constructing the asymptotic solution, it is necessary to determine the values of the derivatives of the unknown functions at point $\tau = \tau_2$, $x = x_2(\tau_2)$ to the second order. There is no need to determine the derivatives of higher orders, since the required accuracy in the numerical solution can be obtained by selecting a sufficiently small initial region. The asymptotic formulas are not given here owing to their cumbersome size.

A program was compiled for the "Strela-4" computer which allows calculating the solution of the problem from beginning to end at one stroke, i.e., during the course of the calculations the program

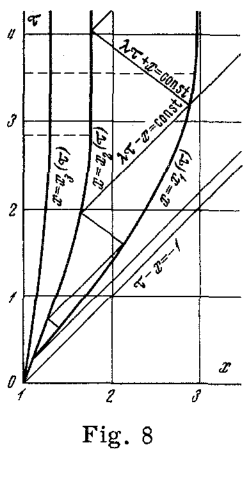

Fig. 8

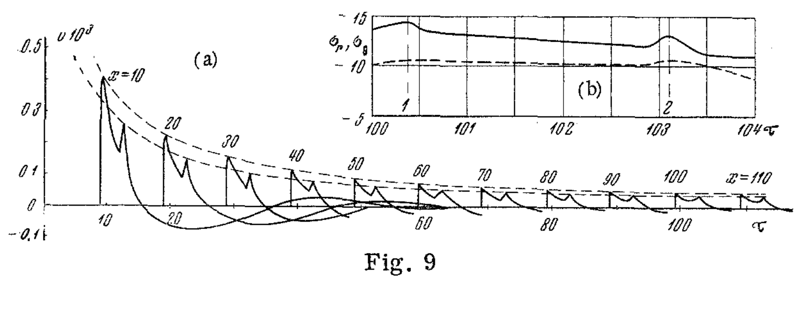

Fig. 9

determines the next stage arising and integrates the system of equations corresponding to it, calculates the asymptotic solutions at the start of the appearance of the third stage, eliminates indeterminates arising in systems (3.3) and (3.8) for the initial instant of the blast, etc. The calculation stops when the unknown functions at the fifth or sixth stages cease to change for the accuracy adopted.

Calculations for certain variants of an explosion in sandstone were performed by means of this program. The solution of the problem of a blast in Plexiglas with the use of the data of [7] (see [5]) was also calculated. The results of the calculations are in good agreement with the experimental data [7].

Figure 8 shows the calculated pattern of the propagation of fracture and expansion fronts in plane $x\tau$ for sandstone for the following initial data [3]:

$$P_0 = 5\cdot 10^3\ \text{atm},\ P_h = 10\ \text{atm} \qquad E = 10^5\ \text{kg/cm}^2$$
$$\sigma = 0.08, \qquad \sigma_* = 30\ \text{kg/cm}^2 \qquad \sigma_{**} = 500\ \text{kg/cm}^2$$
$$k = 1.3, \qquad a = 0.4, \qquad b = 10\ \text{kg/cm}^2, \alpha = 2$$

The first and sixth stages are absent in this case.

Figure 9a shows the time dependence of the mass velocity at various distances from the blast center and Fig. 9b shows the graph of the change of the hoop (dashed line) and radial (solid line) stresses at distance x = 100 for the same initial conditions.

As we see from Fig. 9a, at all distances the mass velocity as a function of time has two maxima: the first corresponds to the appearance of the front of fracture by radial cracks and the second to its exhaustion. The dashed lines show the laws of decay of the mass velocity maxima. Sections 1-1 and 2-2 in Fig. 9b denote respectively the instants of arrival of the elastic waves radiated at the instants of occurrences of the fronts of fracture by radial cracks and of exhaustion of this front.

A comparison of the solution constructed with the solution of the elastic problem shows here, just as for the case of nonporous rocks and Plexiglas, that the elastic wave radiated from the blast center dies out more intensely (for the elastic solution max $|\sigma_r|_{x=100} = 50$ atm and in Fig. 9b max $|\sigma_r|_{x=100} \approx 10$ atm) and has a duration greater by an order than in the elastic solution.

Thus the conclusion made in [5] concerning the character of the effects of fracture and plastic flow in a small region ($x \sim 3$-6) of the blast center on the parameters of the elastic wave radiated at large distances are completely applicable also in the case under consideration.

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